

## Effects of Viscous Damping Models in Earthquake Stress Analysis of Concrete Dams

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### ABSTRACT

Energy dissipation mechanisms are not always explicitly modelled in the time-domain numerical analyses structures. For example, radiation energy losses in the foundation and reservoir sub-domains, during the seismic excitations of a dam, may be simplistically considered by using a viscosity matrix that is proportional to the stiffness matrix of the dam. In the linear seismic analyses of dams, the effects of concrete cracking are often 'simulated' by magnifying the viscosity matrix of concrete. The stress resistance of this imposed viscosity model, that coexists with the material stiffness in the idealized continuum, has not been explicitly investigated in the past finite element analyses of dams. This paper presents comparative results on earthquake induced mechanical and viscous stress responses of a full scale concrete gravity dam. The significant role of viscous damping model in seismic fracture analysis of the dam has been investigated by using a continuum fracture propagation model. Preliminary analyses have shown that the smeared viscosity model may modify the internal stress field as well as the predicted safety against crack initiation and propagation under seismic loads.

### INTRODUCTION

Periodic safety assessment of concrete dams has become a mandatory requirement for the dam owners to ensure public safety and economic stability. Most of the existing concrete dams in Canada were designed and constructed with relatively less stringent criteria for loads and structural performances. As the structural resistances are degrading due to aging effects, many existing dams often fail to satisfy the present safety requirements under revised magnitudes of unusual earthquake and flood loads. Consequently, the practicing engineers are resorting to the use of advanced numerical analysis tools to make a comprehensive prediction of the structural safety of the dams.

The linear frequency-domain seismic analysis method, developed a decade ago (Fenves and Chopra 1984), is often used to predict the tensile stresses at few critical locations of the structure (Fan and Sled 1992). When the predicted stresses considerably exceed the tensile strength of concrete, a transient crack propagation analysis model becomes essential to determine the ultimate safety of the structure. Significant

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efforts have been made in recent years for the development of advanced numerical analysis tools to determine the ultimate resistance of dams by using crack propagation analysis techniques (Saouma et al. 1994, Tinawi et al. 1994). The seismic fracture response of Koyna dam has been simulated by using the continuum fracture (Bhattacharjee and Léger 1993) and damage mechanics (Ghrib and Tinawi 1995) constitutive models. These numerical experiments have indicated that the predicted cracking response may change considerably depending on the use of a constant or a variable stiffness proportional viscous damping matrix.

Mechanical, viscous, and inertia resistances are explicitly considered by three separate property matrices in the time-domain dynamic equilibrium equations. However, no attention is usually given to the inertia and viscous forces during the recuperation of stresses in traditional finite element analysis algorithms. The steady state component of the resistance, idealized by the spring inside a finite element in Fig. 1, is generally compared with the material tensile strength to assess the local safety against crack initiation/propagation. The inertia resistance would be very small for the typical seismic induced strain rates of  $10^{-3}$  to  $10^{-2}$  per sec in concrete dams. However, the effects of viscous damping model could be very significant. To the best of the authors' knowledge, no investigation has been conducted to quantify the viscous stresses in earthquake analyses of concrete dams.

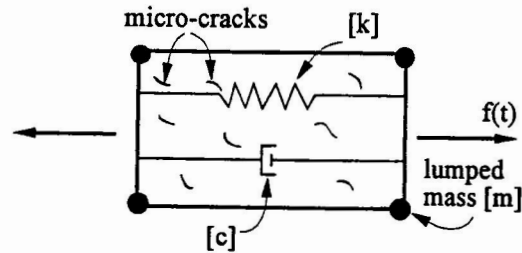


Figure 1: A finite element model.

In this paper, the viscous stress history will be explicitly computed to determine its relative significance in comparison to the mechanical stress response of finite elements. Two different viscous damping models will be considered to investigate the seismic cracking responses of a gravity dam that has been extensively studied by several researchers in the past. The limitations of the present seismic safety assessment procedure, that uses increased damping values in elastic finite element analyses to approximately represent the effects of concrete cracking, will also be discussed.

## NUMERICAL MODELS FOR THE NONLINEAR SEISMIC ANALYSES OF CONCRETE DAMS

The essential aspects of the nonlinear seismic analysis algorithm, (i) solution of dynamic equilibrium equations, (ii) constitutive model for fracture propagation, and (iii) the finite element viscous resistance model, are presented in the following sections. These numerical models have been implemented in the finite element analysis code FRAC\_DAM (Bhattacharjee 1994) to conduct parametric studies.

### SOLUTION OF DYNAMIC EQUILIBRIUM EQUATIONS

The dynamic equilibrium equations of a finite element (Fig. 1) can be expressed as follows:

$$[m] \{\ddot{u}\} + [c] \{\dot{u}\} + [k] \{u\} = \{f(t)\} \quad (1)$$

where  $[m]$  is the lumped mass matrix,  $[c]$  is the viscosity matrix,  $[k]$  is the stiffness matrix,  $\{f(t)\}$  is the vector of applied nodal forces, and  $\{\ddot{u}\}$ ,  $\{\dot{u}\}$  and  $\{u\}$  are relative nodal acceleration, velocity and displacement vectors respectively. The local property matrices and the associated load and response vectors are assembled to obtain the corresponding global dynamic equilibrium equations. The global mass matrix also contains additional coefficients to represent the hydrodynamic interaction effects of the reservoir. The foundation condition has been assumed rigid in the present analyses. The alpha integration method ( $-1/3 \leq \alpha \leq 0$ ) is used to solve the dynamic equilibrium equations. A plane stress model of the dam structure is considered in this paper. The convergence of the incremental time-domain analysis is monitored by examining the global energy balance error (Bhattacharjee and Léger 1993).

#### SMEARED FRACTURE PROPAGATION MODEL

The compressive stress-strain response is considered to be linear elastic. The material stress-strain relationship is assumed to be elastic isotropic at the initial stage. After the major principal tensile stress exceeds the tensile strength,  $\sigma_t$ , a smeared band of cracks is assumed to propagate in the fixed finite element topology. A linear strain softening constitutive model is adopted to conserve the fracture energy,  $G_f$ . The material reference axis system is kept aligned with the principal strain directions, and the shear modulus of the damaged continuum is implicitly defined. Cracks are allowed to open and close under cyclic load effects. No water pressure is, however, considered inside the cracks. The local orthotropic constitutive relations are transformed to obtain the constitutive matrix,  $[D]$ , in global coordinate directions, and the mechanical stresses at a Gauss point,  $\{\sigma\}$ , are calculated from:

$$\{\sigma\} = [D][B]\{u\} \quad (2)$$

where  $[B]$  is the strain-displacement transformation matrix. Detail discussions on the continuum fracture and damage mechanics constitutive models have been documented in Bhattacharjee and Léger (1993) and Ghrib and Tinawi (1995).

#### VISCOUS DAMPING MODEL AND THE CALCULATION OF ASSOCIATED STRESSES

Phenomenological modelling of the material damping behaviour is generally avoided due to the lack of experimental results. Small amplitude forced vibration tests can be conducted to determine the amount of damping on the global vibration modes of dams (Hall 1988). The observed modal damping values include both material damping and wave radiation effects. In numerical analyses of structures, special time-domain boundary conditions (Wolf and Song 1994) or coupled boundary element-finite element models (Wepf et al. 1993, Chandrasher and Humar 1993) may be considered to separate these two effects. In the present numerical analyses, the wave propagation effects are ignored. A stiffness proportional viscous damping matrix is calibrated to provide a specified amount of damping on the initial fundamental mode response of the dam (without considering added mass effects). The use of a lumped mass proportional damping matrix, in the finite element model of a solid continuum, has no practical significance, and is not considered here. The finite element viscous force resistance,  $\{f_v\}$ , and the associated stresses,  $\{\sigma_v\}$ , are thus obtained as follows:

$$\begin{aligned} \{f_v\} &= [c]\{\dot{u}\} = b \int [B]^T [D] [B] dV \{\dot{u}\} = \int [B]^T \{\sigma_v\} dV \\ \text{and} \quad \{\sigma_v\} &= b [D] [B] \{\dot{u}\} \end{aligned} \quad (3)$$

where 'b' is the viscosity parameter. This definition of viscosity matrix is a fictitious representation of the material damping properties, since it is calibrated from the global dynamic characteristics of the dam. The viscous stresses computed from equation (3) are superposed on the mechanical stresses, computed in equation (2), to determine the qualitative influences of the viscous resistance on the element behaviour. However, only the mechanical response is considered to determine the evolution cracks in finite elements.

### SYSTEM ANALYZED

The present investigation on the effects of viscous damping models has been conducted with a finite element model of Koyna dam (Fig. 2(a)). The fundamental period of the dam is  $T_1=0.330$  sec, assuming elastic modulus=31027 MPa, Poisson's ratio=0.2, and mass density=2643 kg/m<sup>3</sup>. Despite a significant phase difference between the dam's fundamental period and the dominant period of the input excitations (Fig. 2(b)), the dam experienced severe cracking at the elevation of the downstream slope change, and eventually has become a bench-mark problem for the verification of seismic analysis models. In the present study, no cracking has been predicted during the static analysis of the dam under self-weight and hydrostatic pressure loads, assuming tensile strength=1.5 MPa and fracture energy=150 N/m. These two parameters have been magnified by a factor of 1.2 during the seismic analyses. The viscosity parameter 'b' has been calibrated to provide 5% damping on the fundamental mode of the dam, without considering the added mass effects. An increased value of damping (10%) has been considered in one of the analyses, discussed in the following section. The time domain integration of the finite element equilibrium equations has been conducted at a step of 0.002 sec, with an algorithmic damping parameter  $\alpha=-0.1$ .

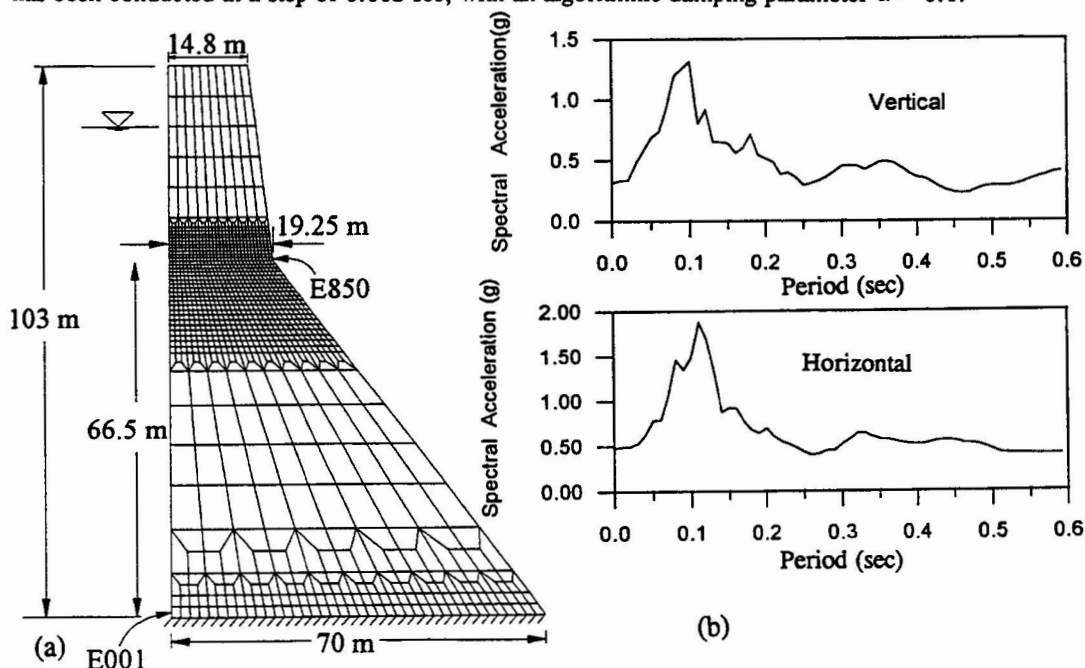


Figure 2: (a) Finite Element model of Koyna dam, and (b) the spectra of Koyna accelerograms.

## LINEAR ELASTIC FINITE ELEMENT ANALYSIS RESULTS

Figure 3(a) compares the principal stress histories of mechanical and mechanical-plus-viscous stress responses of the element E850 on the downstream side (Fig. 2(a)). This location is selected as being the most critical location for crack initiation. It is evident from Fig. 3(a) that the viscous stress component does not cause a significant phase difference between the two stress histories. However, the amplified view of one of the peak stress amplitudes, Fig. 3(b), shows a significant modification of the transient stress resistance. This amplification/reduction of the stress response by the viscous damping model is similar to the strain rate effect, which has not been explicitly considered in the concrete constitutive model. The peak amplitude of the mechanical stress component is, however, about the same as the combined mechanical-plus-viscous stress amplitude. This is due to the fact that, in a homogeneous elastic medium, the velocity wave gradually contributes to the increase of displacement magnitude in successive time steps, and finally decays as the ground acceleration reverses its direction. Based on this elastic finite element analysis result, it seems adequate to consider the peak amplitude of mechanical stress as an index to assess the safety against crack initiation.

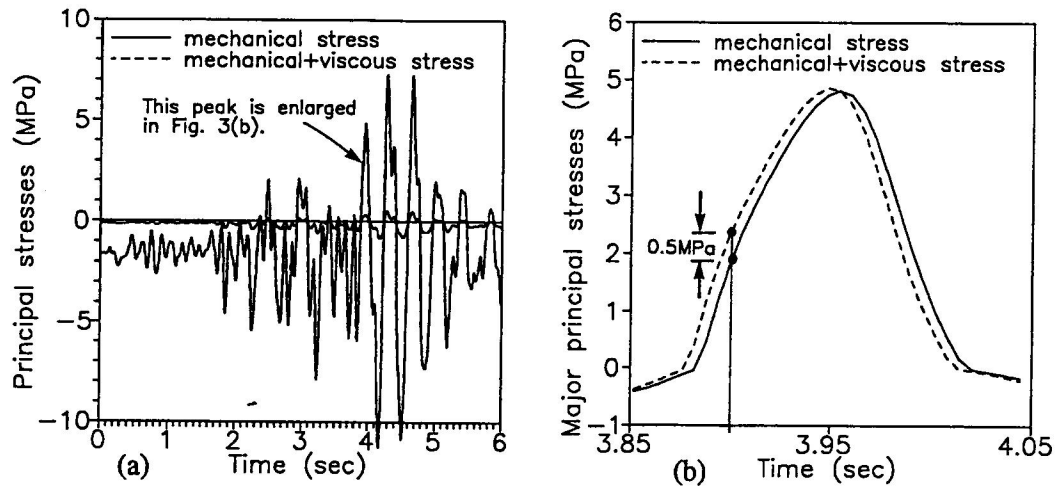


Figure 3: (a) Principal stress time histories of element E850, and (b) a magnified view.

The major principal stress amplitude in this particular analysis has exceeded 7 MPa which is well beyond the tensile strength capacity of concrete. In practical safety assessment of dams, the structural damping value is often increased in an elastic analysis to approximately represent the effects of concrete cracking (Guthrie 1986, Fan and Sled 1992). A subsequent linear seismic analysis, conducted by increasing the modal damping value to 10 %, has not significantly reduced the principal tensile stress in element E850 (not shown here). The state-of-the-practice would require a nonlinear seismic analysis of the dam. The limitations of this linear elastic earthquake safety analysis procedure are discussed in the following section.

## NONLINEAR FINITE ELEMENT ANALYSIS RESULTS

The viscosity parameter 'b' has been calibrated to provide 5% damping in the initial fundamental mode of the dam structure in all nonlinear seismic analyses. The first analysis has been conducted by keeping the viscosity matrix, [c], constant in all finite elements irrespective of the cracking response. Figure 4(a) shows the predicted cracking response of the dam. Very diffused crack profiles are noticed in the top region and at the heel of the dam. The constant viscosity matrix has been tentatively blamed for restraining the crack evolution process in a past investigation (Bhattacharjee and Léger 1993). A quantitative proof is presented in Fig. 4(b), that shows the major principal stress time histories of mechanical and mechanical-plus-viscous stress responses of the cracked concrete element E850. The sudden opening of crack causes a high strain rate in the cracked zone, and the viscosity matrix exerts a high artificial stress resistance to this dynamic opening mode of the crack. The amount of stress transferred across the cracked element by the viscosity matrix is severe enough to diffuse the cracking along the height of the dam. Moreover, the predicted cracking response is less severe than the all-through crack profiles, observed in the actual dam and shaking table tests (Hall 1988).

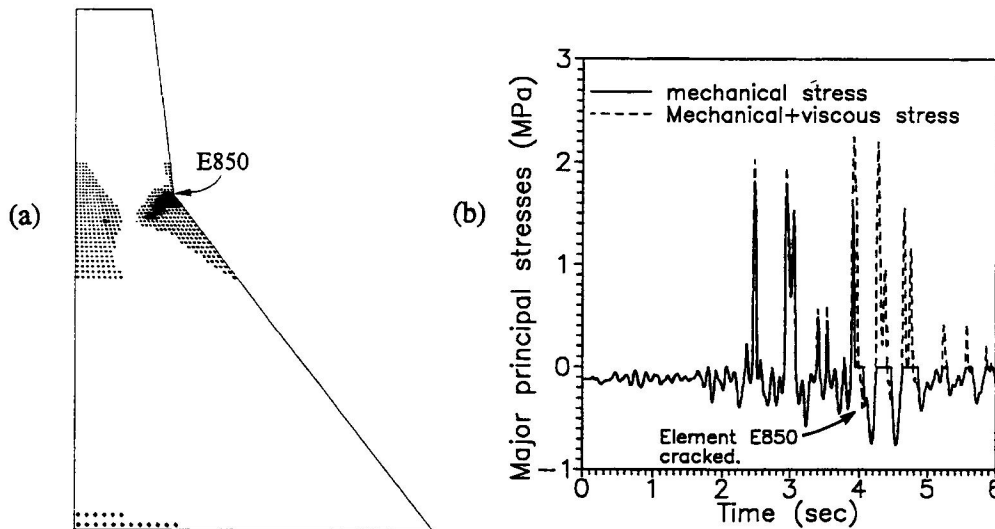


Figure 4: (a) Crack profiles at  $t=4.74$  sec, and (b) the artificial viscous stress resistance of the cracked element E850, assuming [c]=constant.

The constant viscous damping matrix contributes to the numerical stability of the nonlinear finite element analysis algorithm. However, the severity of the artificial stress resistance justifies the modification of the viscous damping matrices in cracked finite elements. In a subsequent analysis the viscosity matrix of a finite element is instantly eliminated,  $[c]=[0]$ , after the initiation of the tensile strain softening process. The predicted crack profiles of this analysis are much localized, as shown in Fig. 5.(a), and resemble the observed field and laboratory responses. The numerical instability, that may arise occasionally during the closing of an undamped finite element with rotating cracked constitutive model, can be effectively eliminated by using a non-zero algorithmic damping parameter, e.g.  $\alpha = -0.1$ .

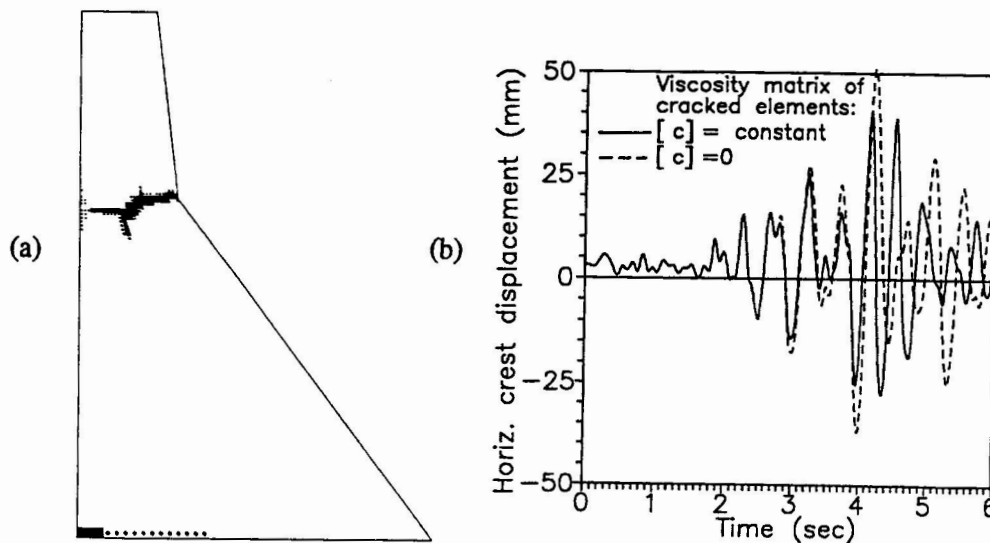


Figure 5: (a) Crack profiles at  $t=3.984$  sec, assuming  $[c]=[0]$  in cracked elements, and (b) time histories of the horizontal displacement at the crest.

The increased severity of cracking in the second nonlinear analysis has considerably modified the dynamic characteristics of the dam, as reflected in the time histories of the horizontal displacement at the crest (Fig. 5(b)). The two nonlinear analyses with different viscous damping models for the cracked elements have obtained essentially same total damping energy dissipations of the dam (not shown here). Moreover, the amount of dissipated energy due to concrete cracking is approximately two orders of magnitude less than the total viscous dissipation. These observations imply that, in simplified elastic seismic analyses, the effects of concrete cracking should be represented by an appropriate modification of the structural properties that directly relate to the integrity of the structure. The energy dissipation due to wave propagation in the adjacent reservoir and foundation media may, however, change due to the changes in structural properties. The wave energy dispersion through the surrounding media should be explicitly represented with advanced time-domain numerical models. A mere modification of the viscous damping coefficient of the dam structure, to represent all these complex issues in the dam safety assessment, seems to be too simple, and the consequences are simply unknown at the present state.

## CONCLUSIONS

The applied dynamic stress is partially resisted by the smeared viscous damping model of a finite element. This should be taken into consideration before the elastic modulus of concrete is arbitrarily magnified in the time-domain seismic response analyses of dams. However, the viscous stress component does not significantly modify the peak magnitude of the stress in linear elastic analyses. This behaviour may depend on the dynamic characteristics of the structure, and on the frequency content of the input motion. Further parametric analyses are required to determine their effects on the predicted response.

If the elastically computed tensile stress exceeds the tensile strength of concrete, nonlinear time-domain analyses should be considered with a concrete crack propagation model. The viscosity matrix must be completely eliminated or be made proportional to the instantaneous stiffness matrix to obtain a conservative cracking response of the structure. Time domain models to represent the dynamic interactions with the reservoir and the foundation may be progressively included in the analysis procedure to investigate the seismic safety of highly critical facilities.

The increase of viscous damping coefficient in the elastic analysis does not represent the effects of concrete cracking. The changes in dynamic characteristics of the structure, after the cracking of concrete, should be considered to develop approximate safety assessment methodologies.

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